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Topology Dimensions of Real Setting Set and the Euclidean Space

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ABSTRACT

Our research project focuses on the study of the concept of dimension, where we explained how this concept arose and developed, and we studied some well-known groups, then we studied this concept in more general topological spaces.

1. Introduction

Topology is a Greek word consisting of two parts (*Topos-Logos*) has been used by mathematician's topology without thinking about concepts and means the study of all structures and components of different spaces and thus summarize the history of topology in a brief and concise way where it began to think about topology through the problem of Euler in the famous issue "Seven Bridges *of Konigsberg*" Euler's paper of 1736 was the first consequence on topological space. The term topology was first introduced to the Germans as *Topologie* in 1847 by Johann Benedict. Then those who specialized in the English language showed that the word "topologist" means every person who specializes in topology.

So, modern topology relies very strongly on the concepts of group theory that was founded by Cantor the late nineteenth century and we will symbolize the family of open groups with the symbol τ_X which represents topology in X.

The concept of dimension was found by Urysohn and Menger, which is that the empty set has a

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dimension (-1). Dimension theory studies the relationship between the nature of a set and the intersection of a number of subsets that contain the set.

The idea of dimension appeared during the concept of dimension in the geometry of a point and level, we symbolize the dimension functionind; Ind; dim found by the researcher *Pears* in 1975 [4]. Dimension functions ind; Ind; dim using s-open sets were studied in 1992 by Al-AbduAllah [1] and dimension functions using b-open sets were studied by Jaber in 2010 [3].

Chapter One (Basic Concepts)

Definition (1-1)

Topology: - To be X a group that is not empty $X \neq \emptyset$. Subset sets are called topology X if the following conditions are met $\tau_X X$

1. $\emptyset, X \in \tau_X$
2. If it is a $\{G_i\}_{i\in I} \in \tau_X$ that $\cup_{i\in I} G_i \in \tau_X$
The union of any family of groups in τ_X be a component also in τ_X.
3. If it is a $\{G_i\}_{i\in I} \in \tau_X$ *that* $\in \tau_x \cap_{i=1}^{n} G_i$

The intersection of a finite number of groups in τ_X be a component also in τ_X

Note (1-1)

Each element in the topology is called an open set.

Note (1-2)

The binary (X, τ_X) is called a topological space and we will refer to this as the symbol X.

Example (1-1)

Let $X = \{a, b, c\}$ that

$\tau_X = \{\emptyset, X, \{a, b\}, \{b\}, \{a\}\}$ topology on X.

Definition (1-2)

Base of Topology: $X, \tau_X X$ is a space of X. The partial of subsets is B from τ_X is a base to topology if each element of topology represents the union of some elements B.

Definition (1-3)

The partial rule of topological space (subbase): - Let (X, τ_X) be a topological space on X. The set of subsets Ϭ is called a partial rule unless τ_X each element of τ_X is a union of finite intersections of some elements of τ_XϬ.

Or put it another way: - If the finite intersections of the elements of Ϭ represent a rule to τ_X .

Definition (1-4)

The interior point: - Let X be a topological space so $A \subset X$ named the point.

$p \in A$ An interior point in A if there is an open set G that contains p a partial of A we symbolize a group the set of interior points with the symbol $A°$.

Definition (5-1)

Exterior point: - Let X be a topological space and that $A \subset X$ the point is named.

$P \in X$ An exterior point to A if there is G an open set containing p and partial from A so that we denote the $G \cap A = \emptyset$ set of exterior points with the symbol $E(A)$.

Definition (1-6)

Boundary point: Let X be a topological space $A \subset X$. The point $p \in X$ is called a boundary point to A if each open group $of\ G$ also contains at least a point of p and contains at least another point of A its complement (complement to the group A) and the set A of boundary points is symbolized by the symbol $b(A)$.

Definition (1-7)

Limit point: - Let X be a topological space and one $A \subset X$. The point $p \in X$ is called a point of purpose unto A if each set G contains p such that $G \cap A \,/\, \{p\} \neq \emptyset$ We symbolize a group the set of A all the end points for the sum of $\acute{A}$.

Definition (1-8)

The closure group *of group* A is defined as the smallest closed group containing A and We symbolize a group the closure group by the symbol $\bar{A}$.

Example (1-2): -

In the usual topology on R

$A = \{x \in R: 0 \leq x \leq 1\}$

The internal points are $A^{\circ} = (0,1)$

The exterior points are

$E(x) = \{x \in R: x < 0\} \cup \{x \in R: X > 1\}$

The boundary points are $b(A) = \{0,1\}$

Example (1-3)

$A = \{a, c, d\}, X = \{a, b, c, d\}$ τ_X $=\{\{\emptyset, X, \{a, b\}, \{b, c\}, \{b\}\}$

$A' = \{a, c, d\}$

Definition (9-1)

Continuity: -Let Y each of X, Y be the topological space be said to be a continuous function $f: X \rightarrow Y$ if the inverse of the image of each set is open in τ_Y an open set in τ_X.

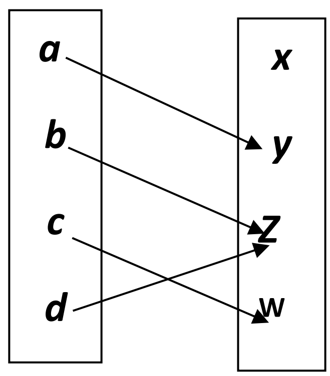
Example (1-4)

Let $Y = \{x, y, z, w), X = \{a, b, c, d\}$

$\tau_X = \{\emptyset, \{a, b\}, \{a\}, \{a, b, c, d\}\}, \tau_Y = \{\emptyset, Y, \{x\}, \{x, y\}, \{y, z, , w\}\}$

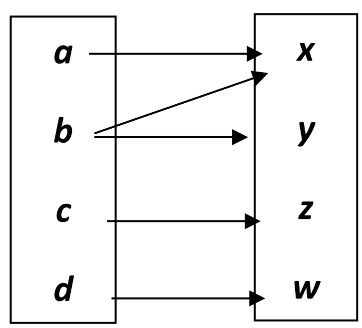
And that the function $f: X \rightarrow Y$ knowledge according to next chart

Clarify that a continuous function f



while g It is not continuous because

$(y,z,w)g^{-1} = \{c, d\} \notin \tau_X$



Definition (1-10)

Topological homeomorphism**:** - Let X, Y be both topological spaces. The function is called

(Topological homeomorphism) if the following conditions are met $f: X \to Y$

1) Continuous f
2) Opposite f
3) Continuous f^{-1}

Then it is said that the two are equivalent topologically X, Y, and it is symbolized by the symbol $X \cong Y$.

Example (1-5)

Let be $(-1,1) = X$ and $Y = R$, let $f: X \to Y$ it be a function defined as follows $f(x) = tan\frac{\pi}{2}x$

It is clear that f the conditions of topological homeomorphism have been met $R \cong (-1,1)$

Definition (1-11)

Regular Space: - A topological space is called X to be regular if each set has a closed F and each $x \notin F$ has two open sets U, V so that

$x \in V$, $F \subset V$, $U \cap V = \emptyset$.

Theorem (1-1)

The topological space is X regular if and only if each set has an open A and every $x \in A$ set has an open set G so that $x \in A \subset \bar{A} \subset G$.

Definition (1-12)

Normal Space: - The topological space X is called a natural space if every two groups are closed F_1, F_2 $(F_1 \cap F_2 = \emptyset)$ *are two open groups in* U, V *so that*

$U \cap V = \emptyset$ $F_1 \subset V$, $F_2 \subset U$.

Theorem (1-2)

The topological space X is normal if and only if each group is F closed and each group is open so that $F \subset G$, there is a group A where $F \subset A \subset \bar{A} \subset G$

Definition (1-13)

Open Function: - The function $f: X \to Y$ is to be an open function if each set is open A in X the f(A) is an open set in Y.

Definition (1-14)

Closed function: - The function $f: X \to Y$ is *to be a closed function if each* set is closed A in X *then* $f(A)$ the *set is closed in* Y.

Definition (1-15)

(Space-T_1): - Topological space X is called a space T_1 if two points $x, y, (x \neq y)$ there are two open groups for every U, V so that each group contains one of the two points and does not contain the other.

That's $y \in V, x \notin V, x \in U, y \notin U$

Definition (1-16)

Let X be a topological space and $A \subseteq X$. It is called the (Feebly Open) Group.

$f - \boldsymbol{open}$ if and only if $A \subseteq A^{\circ - \circ}$

It is called $f - close$ if and only if $\overline{\bar{A}^{\circ}} \subset A$

Note (1-3)

1) Each set is open $f - open$

2) Each set is closed $f - close$

Note (1-4)

The opposite of the previous note is incorrect and we will show this through the following example

Example (1-6)

Let
$X = \{a, b, c, d, e\}, \tau = \{X, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$

Then $f - open$ it is

$X, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}, \{a, b, c, d\}$

We notice that $\{a, b, c, d\}$ is not an open group but it is $f - open$

Theory (1-2)

If it is a X topological space, then τ^f is a topology on X. Where τ^f is the family of all groups $f - open$.

Definition (1-17)

Let X to be a topological space and $A \subseteq X$ and then A to be $f - close$ then boycott all groups $f - close$ in X which it contains A a symbol $\bar{A}^f$.

Theory (1-2)

(1 $\bar{A}^f$ is a closed set

2) $A \subseteq \bar{A}^f$

3) A is set if $f - closed$ and only if $A = \bar{A}^f$

Definition (1-18)

Let X be a topological space and $A \subseteq X$. It's called a group A

Semi-open (s-open) if and only if $A \subseteq \bar{A^{\circ}}$

It is called semi-closed (s-close) if and only if $\bar{A}^{\circ} \subseteq A$

Chapter Two (Dimension Theory)

Definition (2-1)

Dimension: - Dimension is a word used in a variety of meanings, as well as dimension is a separate model as the point has the zero dimension and the curve has the monolithic dimension either the surface has the second dimension.

The dimension is used in mathematics with its branches as well as natural engineering projects, but the concept differs between the two fields. Dimension theory starts from the "dimension function d" and that the function$d$ is defined as a row of topological spaces and $d(X)$ takes an integer or infinity. Where $d(X) = d(Y)$ if it is X, Y a topological equivalence as well as $d(R^n) = n$ for each positive integer.

Definition (2-2)

Let X be a topological space. If $ind\, X = -1$ then $X = \emptyset$ if n the integer is positive or zero, then $ind\, X \leq n$ if the following condition is met

For each $x \in X$ and for each open group G contains $x$and there is an open group U so that

$ind\, b(U) \leq n - 1$ and $x \in U \subseteq G$

If there is no integer n where $ind\, X \leq n$ then $ind\, X = \infty$

Definition (2-3)

We impose X topological space. It is called $f - ind\, X = -1$ then $X = \emptyset$ and if the n integer is positive or zero, we conclude that $f - ind\, X \leq n$ if the following condition is met

For each $x \in X$ and for each open group G contains and x defined on a group $f - openU$ so that $x \in U \subseteq G$ and $f - ind\, X \leq n - 1$

If there is no integer n where for each $f - ind\, X \leq n$ then it $f - ind\, X = \infty$

Theorem (2-1)

Let X be a topological space. So $ind\, X = 0$ if and only if $f - ind\, X = 0$.

Theory (2-2)

Let X be a topological space. If $ind\, X$ is exists, then it is $f - ind\, X \leq ind\, X$.

Theorem (2-2)

Let X be a topological space. If $ind\, X = 0$ and only if $f - ind\, X = 0$.

Theorem (2-3)

Let X be a topological space, if $f - ind\, X = 0$ it is a regular space X.

Proof: -

Let's suppose $x \in X$ and G an open group in X then it is supposed that $x \in G$

$f - ind\, X = 0$ It exists $f - open$ and let A be so, where $x \in A \subseteq G$

$f - ind\, b(A) = -1$ if $b(A) = \emptyset$ to be an open and closed in A

resulting in $x \in A \subseteq \bar{A} \subseteq G$

And that theorem (1-1) we get X that regular space

So X it's a regular space.

Theory (2-2)

In space X it is $f - ind\, X = 0$ realized if and only if the space is not empty and has a base of closed and open groups.

Example (2-1)

Let $X = \{a, b, c, d\}$

and $\tau_X = \{\emptyset, X, \{b\}, \{a, c, d\}\}$ find that the groups $f - open$ are

$\emptyset, X, \{b\}, \{a, c, d\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, d\}\{b, c, d\}, \{a, b$

Since the group $\{a, c, d\}$ is open and closed, then $b\{a, c, d\} = \emptyset, ind\, b\{a, c, d\} = -1$

Since $ind\, X \leq 0$, $X \neq \emptyset$ we get $ind\, X = 0$ from the theory (2-2) .

Theory (2-5)

Let X be a topological space and $A \subseteq X$ then $ind\, A \leq ind\, X$.

Definition (2-4):

Let X be a topological space. So that if $Ind\, X = -1$ then $X = \emptyset$ and if the n integer is positive or zero, we conclude that $Ind\, X \leq n$ if the following conditions are met,

For each closed group F in X and for every open group $F \subseteq G, G$ there is a group $f - open$ and let it be U so that and $F \subseteq U \subseteq G$ and $Ind\, b(U) \leq n - 1$

If there is no integer n, then for $ind\, X \leq n$ we write $Ind\, X = \infty$

Definition (2-5)

Let X be a topological space. So that if $f - Ind\, X = -1$ then $X = \emptyset n$ and if n the integer is positive or zero, then $f - Ind\ X \leq n$ if the following conditions are met

For each closed group F in X and for every open group $F \subseteq G, G$ there is a group $f - open$ and let it be U so that $F \subseteq U \subseteq G$ and $f - Ind\ b(U) \leq n - 1$If there is no integer n then for any $f - Ind\ X \leq n$ and then we write $f - Ind\ X = \infty$.

Theory (2-4)

Let X be a topological space. If it $Ind\ X$ is defined, then $f - Ind\ X \leq Ind\ X$.

Theorem (2-4)

Let X be a topological space. So $Ind\ X = 0$ if and only if $f - ind\ X = 0$

Theorem (2-5):

Let X be a topological space. if $Ind\ X = 0$ it is a natural space X.

Proof: -

We impose F is a closed group in X let U be an open group since $F \subseteq U$ so that

$f - Ind\ X = 0$ there is a group $f - open$ and let A it be, where

$f - Ind\ b(A) = -1$ This means A is open and closed, so that $F \subseteq A \subseteq \bar{A} \subseteq U$

It is the theorem (1-2)

We get X that is a natural space.

Theory (2-5)

In space X it is $f - Ind\ X = 0$ and is realized if the space is not empty and has a topological base so that it consists of closed and open groups.

Example (2-2)

Let's suppose $X = \{a, b, c, d, e\}$

$\tau = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{b, c, d\}, \{b, c, d, e\}, \{b\}, \{b, c\}\}$

$f - open$ the groups are

$\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{b, c, d\}, \{b, c, d, e\}, \{b\}, \{a, b, c, e\}, \{b\}, \{b, c\}, \{a, b, c, d\}, \{b, c, e\},$

Since $\{a\} \subseteq \{a, b\} \subseteq X$ it is realized, so that

$b\{a, b\} = \overline{\{a, b\}} - \{a, b\}^{\circ} = \{a, b, d, e\} - \{a, b, c\} = \{d, e\}$

that the topology on the collection $\{d, e\}$ is discrete. Then $Ind\ b\{d, e\} = -1$ and if $b\{a, b\} = \emptyset$ or $Ind\ b\{a, b\} = \emptyset$ and if $b\{a, b\} \neq \emptyset$

Since space X is not natural, then $Ind\ X \neq \emptyset, Ind\ b\{a, b\} = 0$

So $Ind\ X = 1$ since the $\{a, b\}$ it's a group $f - open$, and so we get $f - Ind\ X = 1$

Theory (2-6)

If X is a topological space. And if X is a space T_1 then $f - ind\ X \leq f - Ind\ X$.

Proof: - The proof will be based on values n .

If it is $n = -1$ then $f - Ind\ X = -1, X = \emptyset$ and so $f - ind\ X = -1$

Suppose the system is true for all $n - 1$. Now let's suppose $f - Ind\ X \leq n, x \in X$ and G an open set so that since $x \in G$, and if X it is a space of T_1 from the hypothesis, then $\{x\}$ it is a closed set $\{x\} \subseteq G$ and $f - Ind\ X \leq n$ then a set $f - open$ then let be V

so that $f - Ind\ b(V) \leq n - 1, \{x\} \subseteq V \subseteq G$ what leads to $f - ind\ b(V) \leq n - 1$

And from that we get $f - ind\ X \leq n$

Example (2-3)

Let
$X = \{a, b, c, d\}, \tau_X = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$

The groups $f - open$ are $\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}, \{a, b\}, \{a, c\},$ $\{b, d\}, \{a, b, d\}, \{b, c, d\}$ the space X is $f - T_1$ but not T_1

Note (2-1)

The example above showed that it is not necessary $f - T_1$ to be T_1 and this condition is not enough, but conditions must be set to become the same, and we will mention the following example, but by putting additional conditions as follows

Example (2-4): -

Let it be X space $f - T_1$ and $f - ind\ X > f - Ind\ X$ because

$f - ind\ X = 1, f - Ind\ X = 0$ This is not a sufficient condition to become T_1

Theorem (2-7)

Let it be X a topological space. If it is X a regular space, then $f - ind\ X \leq f - Ind\ X$

Note (2-2)

Not every space $f - regular$ is X a regular space

Theory (2-7)

Let it be A a closed group and it was $A \subseteq X$ $Ind\ A \leq Ind\ X$

Theorem (2-7)

Let A be a closed and semi-open group and it was $A \subseteq X$ then $f - Ind\ A \leq f - Ind\ X$

Definition (2-6)

We assume X a topological space. So that if $im\ X = -1$ then $X = \emptyset$ and n integer is positive or zero and if $\dim X \leq n$ then each partially open cover $f - open$ is of the order n or ∞ and if there is no such whole number.

Definition (2-7)

We assume X a topological space. So that if was $f - dim\ X = -1$ then $X = \emptyset$ and if n a positive or zero integer $f - \dim X \leq n$ then each set $f - open$ has cover X set has a rank $\leq n$ or ∞ if the number is integer.

***Theorem* (2-9)**

We assume X a topological space. So that if $f - dim\ X = 0$ then X is $f - normal$.

***Theorem* (2-10)**

We assume X a topological space. If each open group is a closed group in X then

$\dim X = 0$ If and only if $f - \dim X = 0$.

***Theory* (2-8)**

If it is $A \subseteq X$ then $\dim A \leq \dim X$

***Theorem* (2-11)**

If it was A the set is open and partially closed from X then $f - \dim A \leq f - \dim X$

Theory **(2-9)**

Let X be a topological space. If every open set is a partial of X is closed then $\dim X \leq f - \dim X$

Theory **(2-10)**

Let X be a topological space. If each open set is closed in X then $\dim X = 0$ if and only if that $f - \dim X = 0$

2. Conclusion

To put it in sum, in the first chapter we included in this study the concepts that we needed to explain, specifically the main concepts in topology, such as the classification of points, continuous functions, topological quivalence, and separation axioms. The concept of the *coc-open* group and the generalization of the axioms of separation, *coc-normal space*, and functions were also discussed. *coc-continuous*, giving axioms and ample examples of the topic.

Therefore, in the second chapter we have focused on the theory of dimension, where we showed what dimension means for groups, including the empty group, the real setting set R and the Euclidean space R^{n}, and then we studied the theory of dimension in general on public spaces.

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