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Utilizing Logical Gates and Boolean Algebra Rules for Qualitative Fault Tree Analysis

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ABSTRACT

In this paper , a various leveled strategy to infer a negligible cut sets from issue tree's is introduced . Since for more convoluted issue trees it won't to be feasible to decide the cut sets by perception so a more organized strategy will be essential. The introduced strategy is represented through two models for various size of shortcoming tree's . Ends are drawn toward the end.

1. Introduction

Issue examination (FTA) is utilized is one as of the significant instruments or exercises among the dependability errands a thing plan. It's anything but a hierarchical deductive examination to show that down occasions prompts the event of the top occasion i.e (the framework disappointment) FTA is likewise helpful for leading a framework wellbeing investigation.

A few specialists introduced various techniques by which an insignificant cut sets can be resolved for various size of frameworks .Some of the methods based on graph theory and Boolean algebra concepts as [12], [14]. Others depend on set theory and probability like [15], [18], or fuzzy set theory. [8], [19].

For the study in question , in the procedure used the logical gates AND and OR with the Boolean algebra rules by considering the events as a binary variables takes the values 0 or 1 to determine the minimal cut sets which equals to zero since they are the main causes of system failure . i.e the top event to occur . An illustrative examples as an applications are presented analysis for system failure with illustrative example.

2. Few Definitions and Concepts: [9],[13],[15]

Definition (1) :Fault Tree

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A shortcoming tree is a diagrainmat portrayal of all conceivable flaw occasions , their sensible blends , and their relationsmp to the framework disappointment.

Definition (2) :Basic Event

An essential occasion is a fundamental initating deficiency occasion that requires no further turn of events . It is represented by circle.

Definition (3) :Intermediate Event

A halfway occasion is a deficiency occasion which happens as a result of at least one predecessor causes acting through rationale entryways .All transitional occasions are represented by square shapes.

Definition (4) :Minimal Cut Set: [17]

A minimal cut set can be defred as the smallest unreducable of basic events required to msure occurrence of . top event i e ,(the system failure) A faule tree will consist of a finite number of minimal cut sets.

3. The Gates of Fault Tree

There are two sorts of shortcoming tree gates:

1. The OR gate which is utilized to show that the yield occasion happens just on the off chance that at least one of the information occasions happen.
2. The AND gate which is utilized to show that the yield flaw happens just if every one of the info shortcomings happen.

The symbols of AND and OR gates are displayed in the accompanying figure separately . for more detail see [2],[5].

(a) AND gate (b) OR gate

4. Boolean Algebra Rules :[3],[4]

The use of Boolean algebra rules in logic design well know . Practically the designer of logic bulid starts with simple basic circuits named logic blocks .All the date is placed as binary signals in a digital computer , hence the algebra of binary numbers becomes very important .In our work we study the probability of a physical system being in binary manners working or fail since the system consists of components each one of it have also two manners and consequently we treat with a system is totally depends on the binary manners of its components . The components are arranged in series and parallel from to get the whole system . The operations of addition , subtraction, multiplication and division are used in ordinary algebra if different from the operations in Boolean algebra.

In our study we concern with the operations AND , OR and inversion AND and OR. Now for the inversion operation rules are

$\bar{0}=1,\bar{1}=0$ where a bar is placed over the one of the two elements i . e 0,1 . For a variable A its inversion is denoted by $\bar{A}$. If at a given time A=1 then $\bar{A}=\bar{1}=0$ .Note In Boolean algebra .

The symbol (.) is used for AND operation .

Then symbol (+) is used for OR operation .

Illustration Example (1):

$$0 . 0 = 0 , \quad 1 .0 = 0 \qquad (1)$$

$0 .1 = 0$, $1 . 1= 1$
$1 . 1 . 1 . \bar{1} = 1 . 1. 1 . 0 = 0$
$1. \bar{0} . 1. 1 = 1. 1 . 1. 1= 1$
$0 + 0 =0$, $1+ 0 = 1$ (2)
$0+ 1=1$, $1 + 1= 1$

Let A , B two variables each one assuming take the values 0 and 1 only

$A. \bar{1} .B =A. 0. B= 0$
$A. 1. \bar{B}. \bar{0}=A . 1. \bar{B}.1$
If A=1 , B = 0
$A. \bar{0}.B.1= 1.1.1.1=1$
$A.1. \bar{B}. \bar{0}=1.1. \bar{1}. \bar{0}$
$=1.1.0.1= 0$

For three variables A ,B and C

A+B+C=A+(B+C)
If A=1 , B=0 , C=0 then
A+B+C= 1 +(0+0)
=1 + 0=1

We list the following operations

Let A, B be two variables each one assuming take a value O or I only. We list the Boolean operations in the following truth table (1) for Boolean operations.

Table (1) for Boolean operations

A	B	A+B	$\overline{A+B}$	A . B	$\bar{A}$	$\bar{B}$	$\bar{A}.\bar{B}$	$\overline{A.B}$	$\bar{A}+\bar{B}$	$\bar{A}$ + B	$A . \bar{B}$
0	0	0	1	0	1	1	1	1	1	1	0
0	1	1	0	0	1	0	0	1	1	1	0
1	0	1	0	0	0	1	0	1	1	0	1
1	1	1	0	1	0	0	0	0	0	1	0

A^'∩A^'=A^'
A∪A=A
1 + 1 = 1

As in figure below:

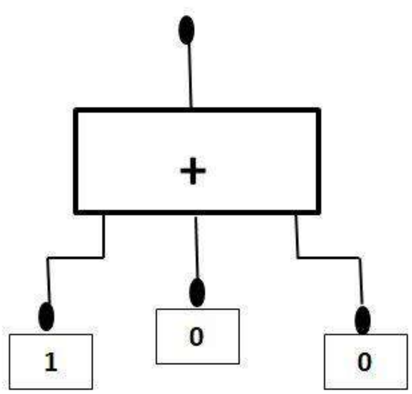


Fig (1) (+) operation

iii. Demorgan Law:

For A and B which are two binary variables.

$\overline{(A+B)}=\bar{A}\bar{B}$ and for more than two variables A, B,C for example

$\overline{(A+B+C)}=\bar{A}\bar{B}\bar{C}$

$\overline{(AB)}=\overline{(A+B)}$ and $\overline{(ABC)}=\bar{A}+\bar{B}+\bar{C}$

5. Illustrative Examples

Example (2) For the following physical structure of water pumping system .

Draw F T and determine minimal cut sets . Talen from [9]

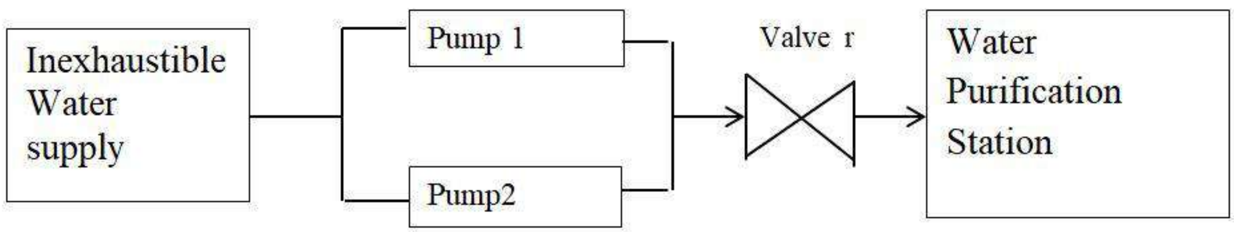


Fig (2) A water supply system

Expect that undesired occasion is there is on progression of water to disposing of the commitment of the line lines , it can demonstrating the framework by the accompanying F T

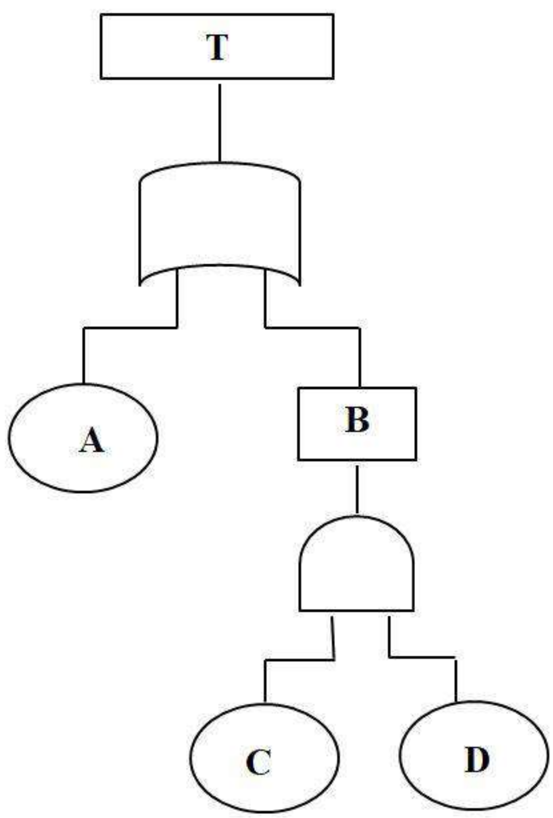


Fig (3) FT of example (2)

Where T the top event = No progression of water

A = valve V comes up short shut
C= pump 1 fails
D= pump 2 fails
The algebraic equation of F T is .
T = A + B , B = C . D
:. T =A + C . D , A,B are binary and B variables taking values 0 or 1 Assume A, B has a value 0
So :
A=0 , B = 0.0
:.T= 0 + 0= 0

Example (3): It is taken from with modify cation A substation is connected to a main electrical network N, three converters C1,C2 and C3 takes their power feeding from the Sub-stations for a high current special machine continuous DC power supply through particular period of the time . The needed power can be made available via a converters . For the sake of the ensure uninterrupted supply C1, C2 and C3 are used if one fail the others stay working.

The physical structure of the system his as in figure below . Draw the FT of the system and write the equation of top event.

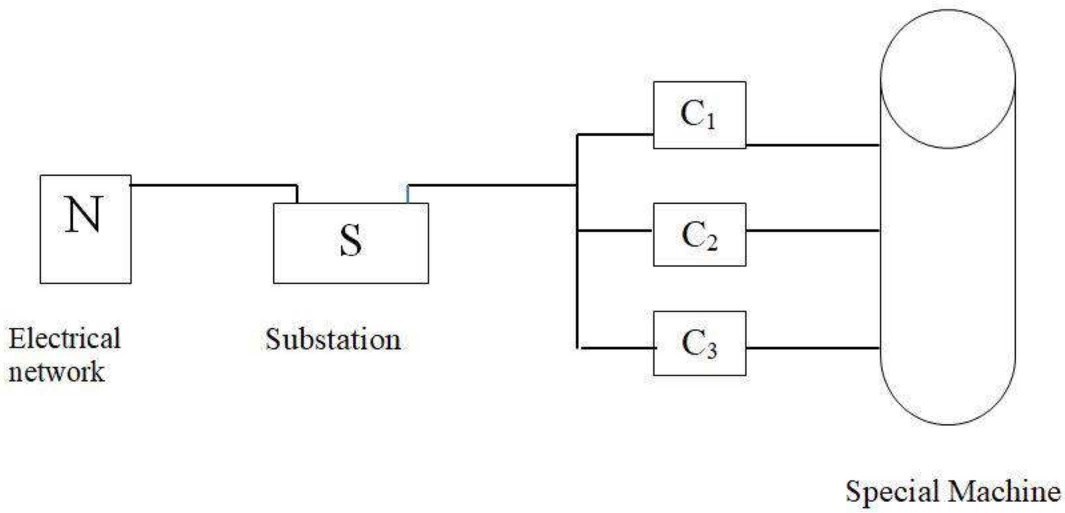


Fig (4) the physical structure of the system

The FT of the physical structure of the system in fig ( ) is as follows .No DC supply to the machine

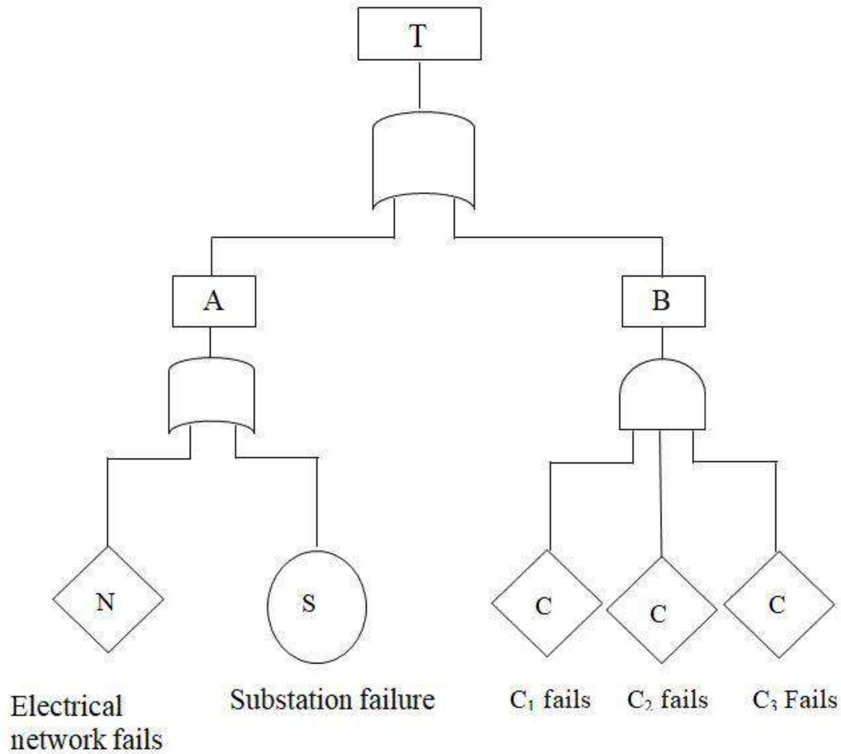


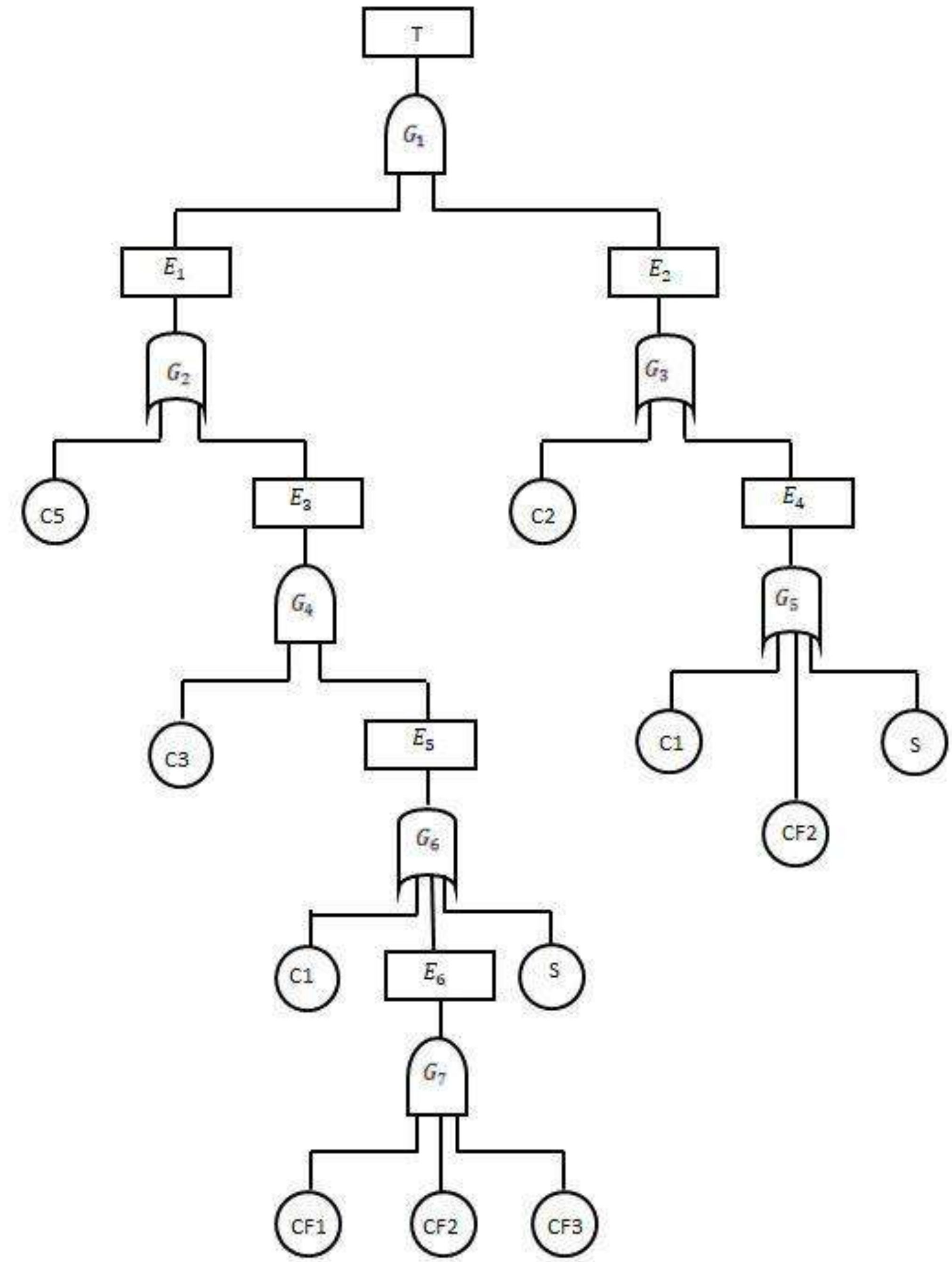
Fig (5) FT of fig (4) where T (the top event) means there is no DC feeding to the machines A means no power supply to C1, C2 ,C3 and B means C1,C2 and C3 all are fail

T = A +B , B = C1. C2 .C3
:. T = N +S + C1. C2 .C3

Notice that N is an event the cause for which cannot specified . The fail are of the converters may be related to other basic events not specified due to defect of data if the failure of C1,C2 and C3 are caused by events E,S .

Example (4):

The failure in the nuclear reactions because of long time running especially the cooling system and temperature control systems which are very important in the nuclear reactions because of the dangerous of disaster of the explosion. Consider the system of jacket water cooling in nuclear reactions as shown in the fault tree below which is taking as problem from [11].



Fig(6): A fault tree of power plant system

Where $G_1, G_2, \ldots, G_7$ are generators

$C_1, C_2, \ldots, C_5$ are cooling units
CF_1, CF_2, and, CF_3 cooling fuel units

And S standby cooling unit

From shortcoming tree above by utilizing the Boolean variable based math tasks to decide insignificant cut sets as follows:-

The top event $T=E_1 . E_2$
$E_1= C_5 + E_3$
$E_2= C_2+ E_4$
$E_3 = C_3 . E_5$
$E_4 = C_1 + CF_2 + S$
$E_5 = C_1 + E_6 + S$
$E_6 = CF_1 . CF_2 . CF_3$
Now by substitution
$T=(C_5+ E_3)(C_2+E_4)$

=(C_5+ C_3 .E_5)(C_2+C_(1)+CF_2+S)
=[(C_5+ C_3 (C_(1)+S+E_(6))](C_2+C_(1)+CF_2+S)
= [C_5+ C_3 {C_1 ┤+├ ⟦(CF⟧_1.⟦CF⟧_(2).CF_3)+S}](C_2+C_(1)+CF_2+S)
By using Boolean rules
T=[C_5+(C_3.C_(1))+C_3.[CF_1.CF_2.CF_3)+(C_3 S)](C_2+C_(1)+CF_2+S)
Noted that C_3.C_(1).C_(1)=C_3.C_(1)
We get the minimal cut sets as in below :-
C_5.C_2,
C_5.C_(1)
C_5.CF_2
C_5.S
C_3.C_(1)
C_3.S
C_3.CF_1.CF_2.CF_3

Noted that
1- Any minimal cut sets causes the failure of the system.
2- Let A =[C_5+ C_3 {C_1 ┤+├ ⟦(CF⟧_1.⟦CF⟧_(2).CF_3)+S}] and
B=(C2+C1+CF2 +S)
A ,B are binary variables Tales the volume 0 or 1.
Assume A, B have a value 0 which means the failure of the system
So: T=0 when
C5. C2 =0 , C5.C1=0 , C5.CF2=0
C5.S =0 , C3.C1=0 and
C3.CF1.CF2.CF3 =0

Example :(5) Assume we have the flaw tree in the figure underneath:

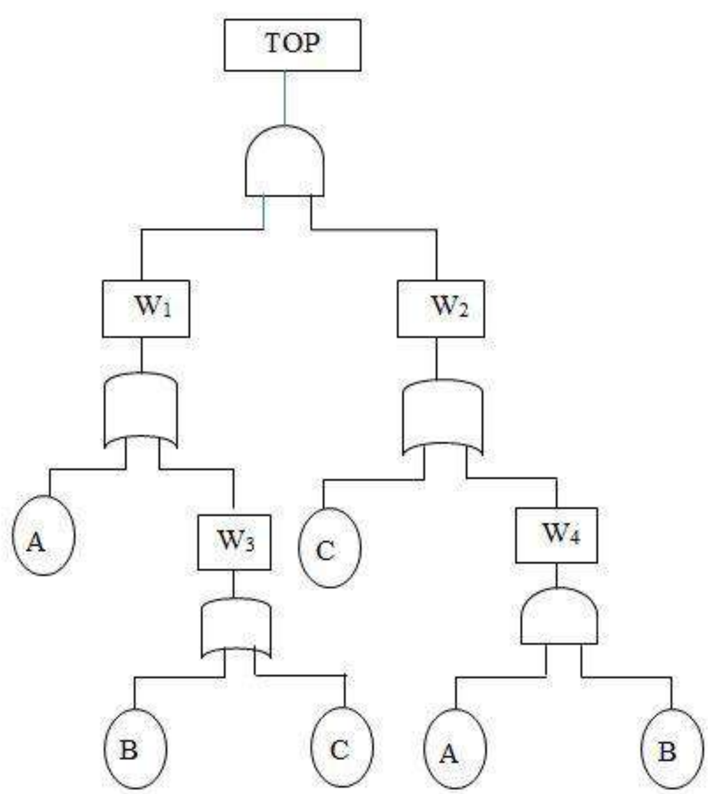


Fig (7) A Fault Tree of a system

We note the equivalent Boolean equation are :

T= W1. W2
W1= A+W3
W2=C+W4
W3=B+C
W4=A . B
There are two type style to solve this system which are:

First type method.
From Top To Down Type[4]
It is starting with top occasion condition and substitute and spread until getting the negligible cut set closure for the top occasion T we have :

T=W1 . W2
=[A+W3][C+W4]
Substitute: W3=B+C
=[A+(B+C)][C+W4]
=A . C+A W4+(B+C). C+(B+C) . W4
=A . C+A . W4+C .B+C . C+B . W4+C. W4
=(A . C+C . B+C+C.E4)+A .W4+B . W4
=C+ A .W4+B . W4
=C+ A(A . B)+(A. B)
=C+ A . B+A . B
=C + A . B

The minimal cut sets are A . B and C. Notes that C =0 or A. B =0 .0 = 0
Causes the system failure

Second type (method)
From Bottom To Up Type
It is start at the base at the tree and squeezed up word with a similar replacement and extension method
The equation having only basic failure

B+C=W3 , W3+A=W1
A+B=W4 , W4+C=W2
W1=A+B+C
W2=C+A . B
Then
W1 . W2=T
(A+B+C) . (C+(A.B)) =T
A . C+ A(A. B)+B .C+B (A.B)+ C. C+C(A.B)=T
A. C+B .C+C+C(A. B)+A .B+ A. B=T
C + A. B= T

We get tow minimal cut sets: A. B and C

Example(6) : Determine the minimal cut sets of a fault tree as shown in figure below:

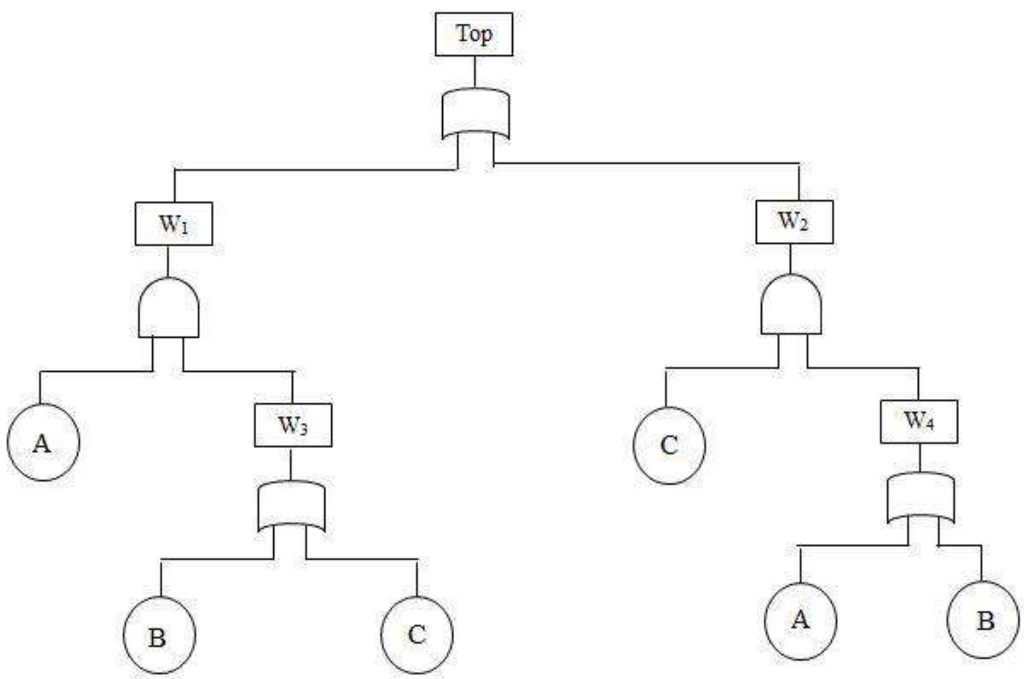


Fig .(8) FT of example

T=W1+W2
E1=A . W3
W2=C . W4
W3= B +C
W4= A + B

:. T=(A .W3)+(C +W4)
=[A . (B+C)]+[C .(A+B)]
=AB+ AC+CA+CB
=AB+AC+BC

The minimal cut sets are AB,AC and BC

Example(7) :Derive the equation of the top event

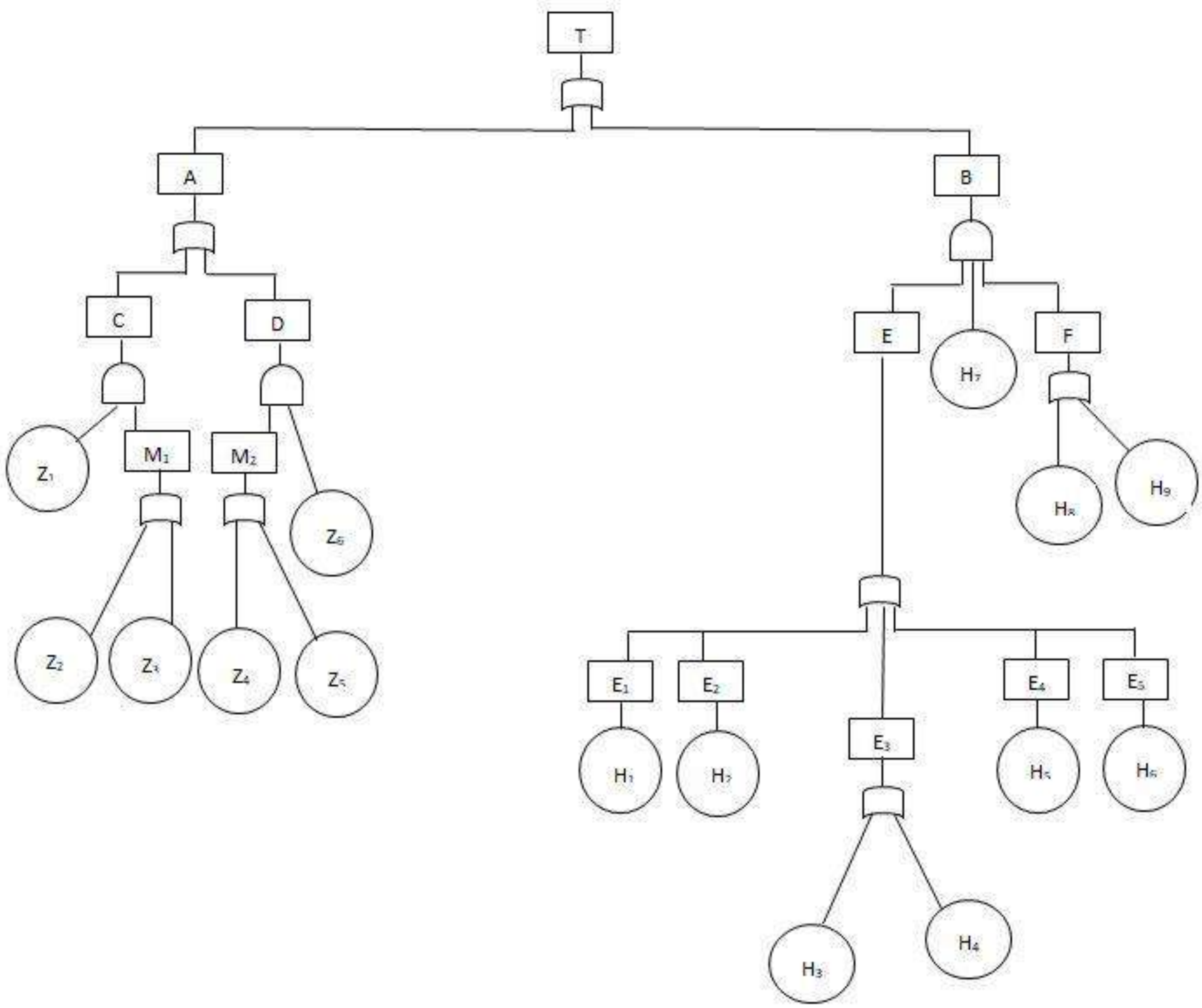


Fig.(9) for FT of example (7)

Solution:

We have
T=A+B
Where A= C + D and B= E . F
By substitution in T
T = (C +D) +(E . F)

Since C = Z1 . (Z2 + Z3) , D = (Z4+ Z5) . Z6
And E = E1+E2+E3+E4+E5 , F = H8 + H9
:. T = [Z1 .(Z2+Z3)+(Z4+Z5).Z6]+[(H1+H2+H3+H4+H5+H6) .H7.(H8+H9)]

Example(8) : Determine minimal cut sets for the fault tree below using Boolean algebra laws.

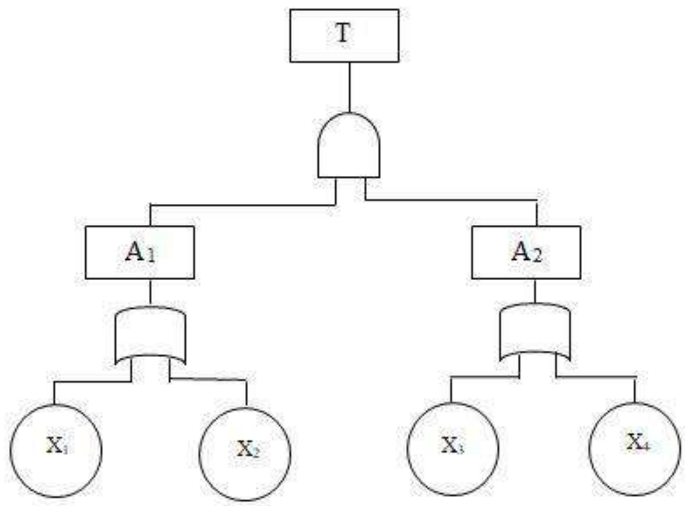


Fig (10) a Fault tree A1 , A2 are in termediate events X1 , X2 , X3, X4 are basic events

A1=X1 U X2 or A1= X1 +X2
A2=X3 U X4 or A2= X3 +X4
The occurrence of top event T occure when A1 and A2 occur
T= A1 . A2
=(X1 +X2) . (X3 +X4)
=(X1X3+X1X4+X2X3+X2X4)
By using Boolean algebra laws :
A1 , A2 are binary variables taking value 0 or 1
Assume A1, A2 has a value 0 indication failure so :
A1=X1 + X2
= 0 + 0 = 0
A2 =X3 + X4
=0 + 0= 0
Hence T=A1 + A2
=0 . 0 = 0
According to Boolean Laws
T=0 only when
X1X3 =0 , X1X4= 0, X2X3=0 and X2X4=0
So the minimal cut sets are :
X1X3, X1X4, X2X3 , X2X4
Second method :
T= A1 + A2
=(X1+ X2) . (X3 + X4)
=X1X2+ X1X4+ X2X3+ X2X4
Pr(T)=Pr[X1(X3+X4) + X2 (X3 + X4)]
=Pr[(X1 +X2) . (X3+ X4)]
=Pr(X1 + X2) Pr(X3 + X4)
=(PrX1+PrX2-PrX1X2)(PrX3 + PrX4-PrX3X4)

6. Conclusions

The suggested method is easier for technologists and engineers by using the digitals in application

The method deals with the qualitative analysis for any fault tree via determining the minimal cut sets as a main causes for system failure.

From the final events expression it is easily to calculate the system reliability R(S) from the system failure F(S) and R(S) where

R(S) = 1 - F(S)

Since the failure system occurs when one of the minimal cuts occurs .

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